

RESONANT AMPLIFICATION OF INSTABILITY WAVES IN QUASI-SUBHARMONIC TRIPLETS WITH FREQUENCY AND WAVENUMBER DETUNINGS

V.I. Borodulin, Y.S. Kachanov, D.B. Koptsev, and A.P. Roschektayev

Institute of Theoretical and Applied Mechanics SB RAS,
630090 Novosibirsk, Russia

1. Introduction

Probably the first experimental evidence of an important role of the subharmonic resonance in the process of the laminar-turbulent transition in adverse pressure gradient (APG) boundary layers was obtained in [1]. Until recently, the only available direct experimental study of the subharmonic resonant interactions in the APG boundary layer was performed in [2] for rather small values of the Hartree parameter ($\beta_H = -0.06$ and -0.09). In agreement with the theoretical prediction, a rapid growth and subsequent saturation of subharmonic amplitudes was found under controlled disturbance conditions. However, the double-exponential growth predicted by theory has not been found, and resonant interactions with frequency and wavenumber detunings have not been examined. Some new results were obtained in [3] where all main properties of tuned resonances were studied for a moderate Hartree parameter $\beta_H = -0.115$. Optimal parameters of resonant triplets were found, a strong dependence of the interaction on phase relationships was examined, and the double-exponential growth was discovered. However, many questions are still open.

The goal of the present experiments was to narrow the gaps existing in this area. In particular, we aimed to study the resonant interactions in the presence of frequency and spanwise-wavenumber detunings for quasi-subharmonic modes.

2. Experimental Procedure

Experiments were conducted in the closed-loop low-turbulence wind tunnel T-324 of ITAM at the free-stream velocity $U_\infty \approx 9$ m/s. This wind tunnel has a 4 m long test section with a 1×1 m cross-section. The free-stream turbulence level at the present experimental conditions was below 0.02% in the frequency range above 1 Hz.

In general, the experimental setup was similar to that used in [3, 4]. The boundary layer under investigation developed on a flat plate installed horizontally in the wind-tunnel test section under a zero attack angle and was equipped with a flap to provide the possibility to control the local attack angle of the flow in the vicinity of the plate leading edge. The APG was induced over the plate with the help of an adjustable wall-bump mounted on the test-section ceiling. It was shown that the basic potential flow and the flow inside the boundary layer are two-dimensional (within an experimental accuracy) and correspond very well to those calculated for Hartree parameter $\beta_H = -0.115$.

The present experiments were performed at controlled disturbance conditions. Different kinds of perturbations were excited in the boundary layer by means of a universal disturbance source VS-II developed in [5] (see also [3]) and modified for the purposes of the present experiments. Three groups of regimes were investigated:

- Main Resonance (MR) studied in [3] was reproduced again with a 2D initial wave of a relatively high amplitude (frequency $f_1 = 109.1$ Hz, which is close to the most linearly

Report Documentation Page

Report Date 23 Aug 2002	Report Type N/A	Dates Covered (from... to) -
Title and Subtitle Resonant Amplification of Instability Waves in Quasi-Subharmonic Triplets With Frequency and Wavenumber Detunings		Contract Number
		Grant Number
		Program Element Number
Author(s)		Project Number
		Task Number
		Work Unit Number
Performing Organization Name(s) and Address(es) Institute of Theoretical and Applied Mechanics Institutskaya 4/1 Novosibirsk 530090 Russia		Performing Organization Report Number
Sponsoring/Monitoring Agency Name(s) and Address(es) EOARD PSC 802 Box 14 FPO 09499-0014		Sponsor/Monitor's Acronym(s)
		Sponsor/Monitor's Report Number(s)
Distribution/Availability Statement Approved for public release, distribution unlimited		
Supplementary Notes See also ADM001433, Conference held International Conference on Methods of Aerophysical Research (11th) Held in Novosibirsk, Russia on 1-7 Jul 2002		
Abstract		
Subject Terms		
Report Classification unclassified	Classification of this page unclassified	
Classification of Abstract unclassified	Limitation of Abstract UU	
Number of Pages 6		

unstable one) and a pair of oblique 3D subharmonics (frequency $f_{1/2} = f_1/2$, optimal for resonance spanwise wavenumbers $\beta_R = \pm 0.131$ rad/mm) with relatively small amplitudes;

- Main Resonances with frequency Detunings (MRD) with the same 2D fundamental wave and a pair of 3D quasi-subharmonics (with the same spanwise wavenumbers as in regime MR) at frequencies $f = f_{1/2} + \Delta f$ (where Δf was varied from $-0.9f_{1/2}$ to $+0.9f_{1/2}$).
- Main Resonances with Wavenumber Detunings (MRWD), where 4 different values of the subharmonic spanwise wavenumber β (at frequency $f_{1/2}$) were tested in addition to the resonant value β_R . They covered a range from 0 to $3\beta_R$.

The streamwise velocity component was measured by a hot-wire and subjected to ensemble averaging (using the disturbance generator as a reference) and filtering in Fourier space.

Axis x was directed downstream, z is the spanwise axis. Disturbance source was located at $x = 300$ mm (zero is at the leading edge). Spanwise position $z = 500$ mm corresponds to the experimental model centre.

3. Frequency Detunings

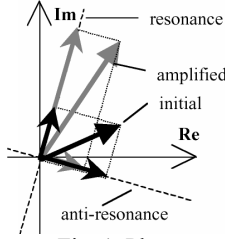


Fig. 1. Phase synchronisation.

For the correct interpretation of the experimental results it is necessary to outline an important property of the subharmonic resonance. Namely, there is an optimal phase shift between the subharmonic pair and the fundamental wave [3]. The resonant mechanism intensifies the disturbance growth when the phase shift is optimal and suppresses it when it is orthogonal to the optimal one. This point is illustrated in Fig. 1 where both initial and amplified subharmonic vectors are shown decomposed into resonant and anti-resonant components. As the result, the resonant interaction should provide a phase synchronisation between the fundamental wave and the subharmonic pair.

The typical downstream evolution of the frequency spectra in regime MRD is shown in Fig. 2 for frequency detuning $\Delta f = -0.1f_{1/2}$. Initially ($x = 350$ mm) there are two peaks in the spectrum, one is for the fundamental frequency f_1 and another — for quasi-subharmonic oblique waves with frequency $f_{1/2} + \Delta f$. Farther downstream the resonant interaction produces a new peak at frequency $f_{1/2} - \Delta f$ (the local ‘symmetrisation’ of the spectrum with respect to the subharmonic frequency). Additional peaks appear at frequencies $(2n+1)f_{1/2} \pm \Delta f$, $n = 1, 2, 3, \dots$. This behaviour is similar to that observed in [6] in the zero pressure gradient case.

As far as there is no any definite phase relationship between the detuned subharmonic modes and the fundamental wave, let us introduce a notion of an ‘*effective mode*’ that is convenient for subsequent analysis (Fig. 3). In this figure ‘left’ and ‘right’ time traces $u'(t)$ corresponds to spectral peaks located to the left and right from the exact subharmonic

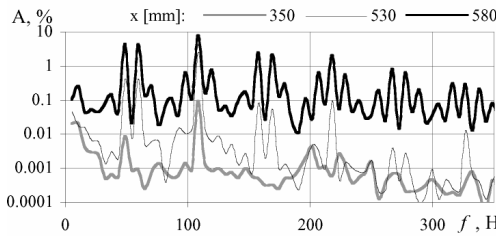


Fig. 2. Downstream evolution of frequency spectra in regime MRD ($\Delta f = -0.1f_{1/2}$)

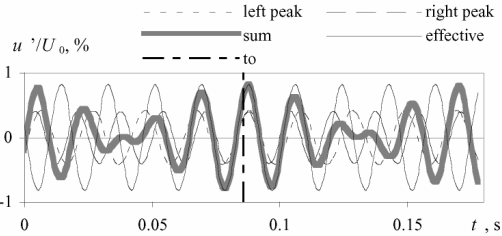


Fig. 3. Definition of effective subharmonic.

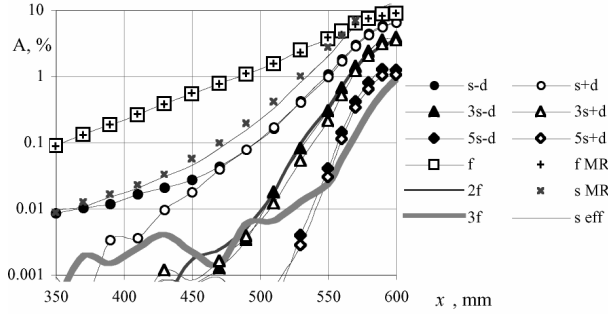


Fig. 4. Amplification curves. Notation: s — subharmonic, f — fundamental, d — detuning, eff — effective. Regime MRD.

frequency. Their sum represents a subharmonic wave with frequency $f_{1/2}$ and with the amplitude modulated in time. Its maximum appears when both quasi-subharmonics have phases optimal for interaction with the fundamental wave (one of these time moments is designated in Fig. 3 as t_0). The ‘effective subharmonic’ is the wave, which coincides with the ‘sum’ mode at the moment t_0 but has constant amplitude (and phase). It is easy to show that the amplitude of the effective subharmonic is equal to sum of amplitudes of left and right quasi-

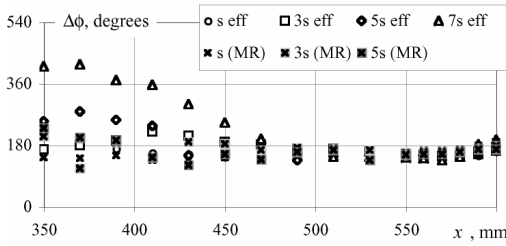


Fig. 5. Phase difference between amplified modes and fundamental wave (degrees of fundamental wave). See notation in Fig. 4. Regime MRD.

Figure 4 shows amplification curves of harmonics $(2n+1)f_{1/2} \pm \Delta f$ ($n = 0, 1, 2$), kf_1 ($k = 1, 2, 3$), and the effective subharmonic in case of a small frequency detuning ($\Delta f = -0.1f_{1/2}$) together with the curves obtained for tuned resonance (regime MR). The fundamental wave grows exponentially and identically in both regimes (MR and MRD). Modes $(2n+1)f_{1/2} \pm \Delta f$ demonstrates the double-exponential amplification within a certain range of the x -coordinate with a subsequent saturation. The behaviour of the effective subharmonic indicates that the mechanisms of amplification are the same in

both regimes. A satisfaction of the phase synchronism condition $\Delta\phi = \phi_{1/2} - \phi_1 = \text{const.}$ corroborates this conclusion (Fig. 5). Indeed, the phase synchronisation appears gradually for all studied modes and the resonant phase shift $\Delta\phi$ is the same in MRD and MR regimes.

Figure 6 shows amplification coefficients for large frequency detunings (in the range $\Delta f = \pm 0.9f_{1/2}$). Similar to the Blasius case, the resonance has a large width in the frequency spectrum but, in contrast to this case, the maximum growth is observed not at exact subharmonic frequency but at detunings of about +30%. This phenomenon has no simple explanation at present. The phase synchronisation for large frequency detunings is shown in Figs. 7 to 9. Despite both excited and ‘mirror’ (symmetric) modes have different phases in the region of their rapid growth ($x = 530$ mm), the effective mode have a constant phase (Fig. 7) and remains synchronised with the fundamental wave (Fig. 8). The same is true for the effective modes based on $3f_{1/2} \pm \Delta f$ and $5f_{1/2} \pm \Delta f$ (Fig. 9).

4. Spanwise-Wavenumber Detunings

The absolute value of the optimal (for the resonance) spanwise wavenumber found in [3] is equal to 0.131 rad/mm that corresponds to wavelength $\lambda_z = 48$ mm. To investigate the effect of the wavenumber detunings, four regimes were studied with excitation of subharmonic pairs with wavenumbers $\beta = \pm 0.393, \pm 0.196, \pm 0.098$ and 0 rad/mm, which correspond to $\lambda_z = 16, 32, 64$ mm, and infinity (the latter mode is two-dimensional). Three examples of spanwise

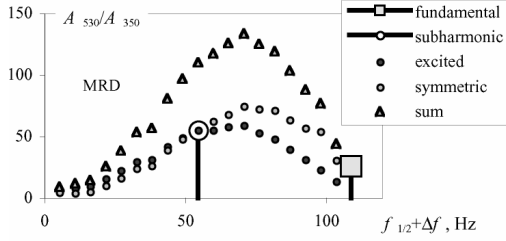


Fig. 6. Amplification coefficients of quasi-subharmonic modes vs. excitation frequency. MRD.

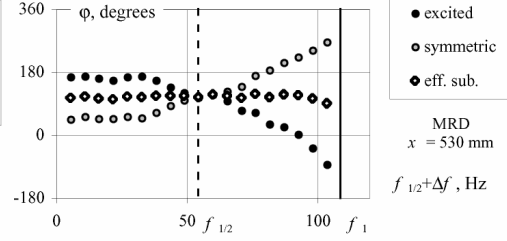


Fig. 7. Phases of quasi-subharmonic modes vs. excitation frequency. Regime MRD.

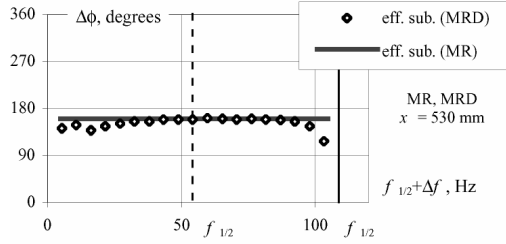


Fig. 8. Phase shift between quasi-subharmonic modes and fundamental wave vs. excitation frequency. Regime MRD.

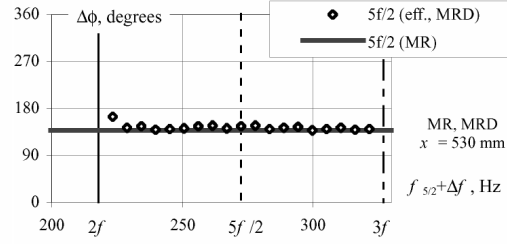


Fig. 9. Phase shift between effective mode $5f_{1/2}$ and fundamental wave vs. excitation frequency. Regime MRD.

distributions of the subharmonic amplitudes and phases are given in Fig. 10 for three values of β for $x = 510$ mm. They are typical for standing waves produced by superposition of oblique wave-pairs propagating along the z -axis in opposite directions.

During the resonant interaction the fundamental wave amplitude grows downstream exponentially (independently of β), while the subharmonics demonstrate a more rapid amplification, which depends on β dramatically (Fig. 11). Several subharmonic amplification curves are given in Fig. 12 in the double-logarithmic scale. As one can see, the subharmonic amplitudes increase in a linear way (in this scale) in ranges of their most rapid growth. This double-exponential behaviour was predicted by theory for tuned resonances but never observed for wavenumber detuned cases. In the range up to $x \approx 500$ mm the amplitude evolution of mode with $\beta = 0.393$ rad/mm is quite different from others (Fig. 11). The 2D subharmonic mode growth is much slower compared to 3D ones. However, approximately the same behaviour is observed at this stage when the fundamental mode is switched off. In absence of the fundamental wave the growth of all subharmonics was rather slow and mainly exponential one.

In MRWD regimes the phase synchronisation was observed similar to regimes MR and MRD (Fig. 13). In these plots the phase shifts $\Delta\phi$ between the subharmonic pairs and the fundamental mode are shown for two cases in each presented regime. The solid symbols correspond to regimes with simultaneous excitation of the subharmonic and fundamental modes, while the open symbols were obtained at their separate excitation, i.e., when these waves developed independently of each other. In case of tuned resonance (regime MR, $\beta = 0.131$ rad/mm, Fig. 13*right*) the subharmonic mode was phase locked (i.e., in the synchronism) with the fundamental one in the whole studied streamwise range and the resonant phase shift is close to 180° similar to the case of frequency detuned resonances (Fig. 5). Its ‘natural’ behaviour (in absence of the fundamental wave) is the same in the beginning (because this is tuned resonance) but different in the end due to change of the basic-flow parameters (and the disturbance frequency parameters). The synchronism of the subharmonics with smaller wavenumber ($\beta = 0.098$) is also present. Despite it gets worse fare downstream compared to the case of tuned resonance, the phase difference of the ‘naturally’ developing subharmonic

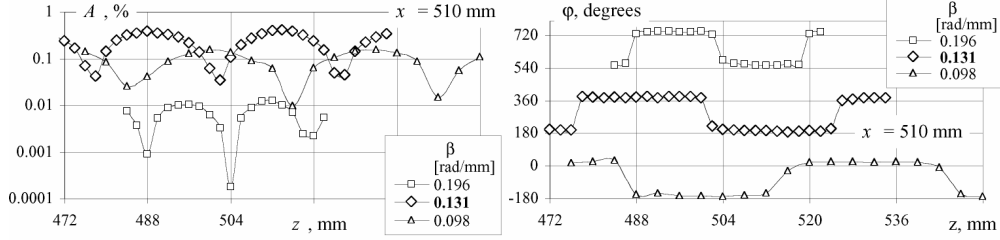


Fig. 10. Spanwise distributions of amplitudes (left) and phases (right) of subharmonics in case of tuned ($\beta = 0.131$ rad/mm) and detuned resonance. Regime MRWD.

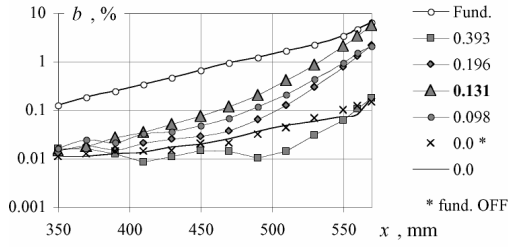


Fig. 11. Downstream evolution of disturbances at fundamental and subharmonic frequencies with different spanwise wavenumbers. MRWD.

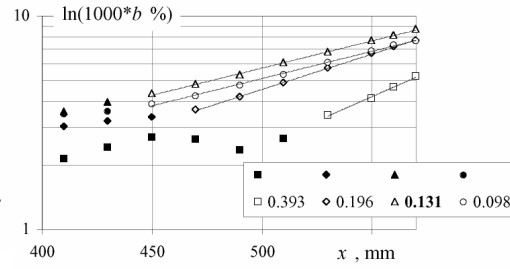


Fig. 12. Double-exponential growth of disturbances at subharmonic frequencies. Regime MRWD.

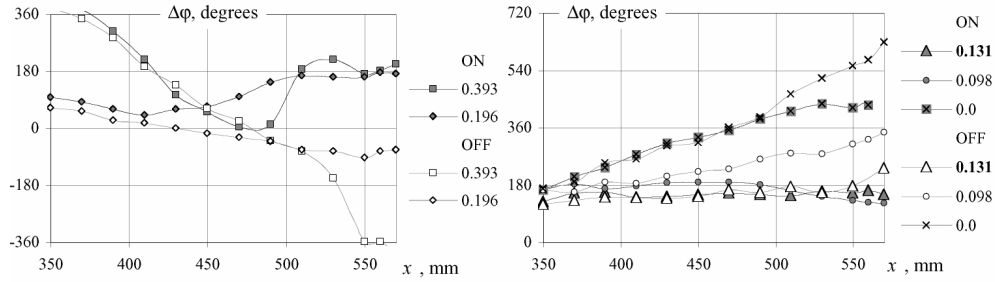


Fig. 13. Phase difference between fundamental and subharmonic modes in case of their simultaneous (resonance ON, regime MRWD) and separate (resonance OFF) excitation.

Left — large spanwise wavenumbers; right — resonant ($\beta = 0.131$ rad/mm) and low wavenumbers.

deviates stronger from the resonant value in the end of the studied range. As to the 2D subharmonic ($\beta = 0$), it is not synchronised at all and not amplified by the resonance (see Fig. 11). For positive wavenumber detunings (Fig. 13/*left*) the phase synchronisation is not observed in the beginning but it appears in the streamwise ranges where the resonance starts to amplify the subharmonics (see Fig. 11). This results is explained by growth of the fundamental wave and, consequently, by a downstream enhancement of the resonant interaction, which leads to an extension of the spanwise wavenumber range of the resonantly amplified subharmonics.

The dependence of the subharmonic amplification on its spanwise wavenumber is summarised in Figs. 14 and 15. Figure 14 shows the integral subharmonic amplification factors for all studied wavenumbers and their downstream evolution. It is seen that at all stages of evolution the largest subharmonic amplification is observed for the tuned resonance (regime MR, $\beta = 0.131$ rad/mm), while for higher and lower spanwise wavenumbers the growth factors are lower, in contrast to the case with frequency detunings (Fig. 6). Figure 16 shows local subharmonic increments determined approximately in the centre of the interaction region ($x = 530$ mm) in regimes with and without interaction (i.e. in presence and in absence of the

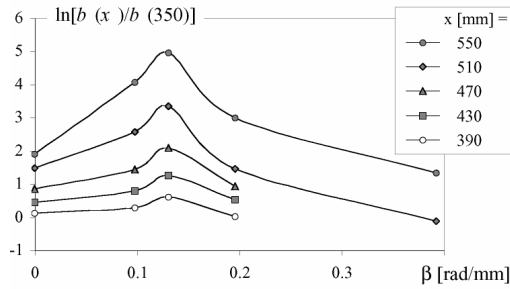


Fig. 14. Subharmonic amplification coefficients for different downstream locations and spanwise wavenumbers. Regime MRWD.

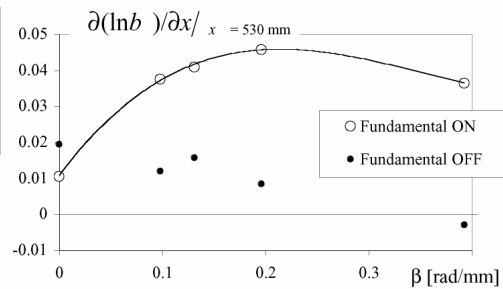


Fig. 15. Growing rates of subharmonics with different spanwise wavenumbers in the region of double-exponential amplification ($x = 530$ mm).

fundamental wave). It is seen that the resonance enhances significantly the increments for all studied subharmonic wavenumbers excluding the 2D subharmonic case ($\beta = 0$). Both plots (Figs. 14 and 15) demonstrate the important property of studied resonance: it takes place in a broad band of values of the subharmonic spanwise wavenumber.

5. Conclusions

- The mechanisms of resonant growth of quasi-subharmonic TS-waves in an APG boundary layer is studied in a wide range of frequency and spanwise-wavenumber detunings.
- It is found that in cases of detuned resonances these mechanisms are the same as in case of tuned resonance and very similar to those studied previously in the Blasius basic flow.
- The double-exponential resonant amplification of 3D quasi-subharmonic waves is found in presence of 2D fundamental wave in wide ranges of frequency and wavenumber detunings.
- In streamwise ranges where this amplifications takes place the resonance phase synchronism condition is found to be satisfied in frequency ranges around $f_{1/2}$, $3f_{1/2}$ and $5f_{1/2}$ and in a wide spanwise-wavenumber range (for the exact subharmonic frequency).
- The resonant growth of the wavenumber detuned subharmonics is found to be always weaker than in the tuned case. Meanwhile, it turned out that the resonant amplification is greater for frequency detuned subharmonics with positive detunings compared to the tuned resonance (in contrast to the Blasius case).

This work was supported by the Russian Foundation for Basic Research (Grant No. 00-01-00835).

REFERENCES

1. Wubben F.J.M., Passchier D.M., Van Ingen J.L. Experimental investigation of Tollmien-Schlichting instability and transition in similar boundary layer flow in an adverse pressure gradient // *Laminar-Turbulent Transition* / Eds. D. Arnal, R. Michel. Berlin: Springer-Verlag, 1990. P. 31-42.
2. Corke T., Gruber S. Resonant growth of three-dimensional modes in Falkner-Scan boundary layers with adverse pressure gradients // *J. Fluid Mech.* 1996. Vol. 320. P. 211-233.
3. Borodulin V.I., Kachanov Y.S., Koptsev D.B. Study of resonant instability wave interaction in self-similar boundary layer with adverse pressure gradient // *Intern. Conf. on Methods of Aerophysical Research: Proc. Pt 1.* Novosibirsk: Publishing House of Siberian Branch of RAS, 2000. P. 47-52.
4. Kachanov Y.S., Koptsev D.B. Three-dimensional stability of self-similar boundary layer with a negative Hartree parameter. 1. Wave-trains // *Thermophysics and Aeromechanics*. 1999. Vol. 6, No. 4. P. 443-456.
5. Borodulin V.I., Gaponenko V.R., Kachanov Y.S. Method of introduction of normal instability modes into the 3D boundary layer // *Intern. Conf. on Methods of Aerophysical Research: Proc. Pt 2.* Novosibirsk: Inst. Theor. & Appl. Mech., 1996. P. 39-45.
6. Kachanov Y.S., Levchenko V.Y. The resonant interaction of disturbances at laminar-turbulent transition in a boundary layer // *J. Fluid Mech.* 1982. Vol. 138. P. 209-247.